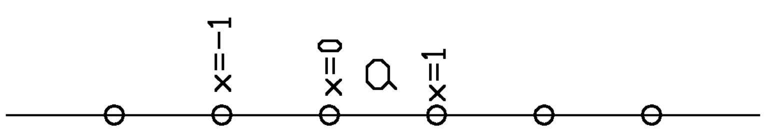
Cosmology, PHY 28
Professor Susskind
Session 1, January 12, 2009

<u>Summary of Concepts</u>
Geometry of Expanding Universe
Cosmological Principle
Hubble's Law
Hubble's Constant
Expanding Closed Space

Consider a line along which are galaxies equally spaced a distance of *a* apart. The Cosmological Principle is that space is homogeneous and isotropic (on a scale of hundreds of millions of light years), which we will assume is valid throughout this course. Label each galaxy by its position from an arbitrarily chosen initial one (x = 0, x = -1, x = +1 etc…). The distance (*D*) between two galaxies is given by

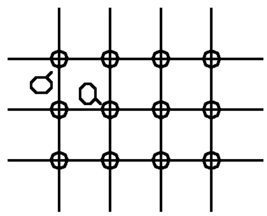
$D = a \cdot \Delta x$, where a is called the scale factor and Δx is an index.



Suppose now two giants at each end of the line pull on it to stretch it away from the origin, whereby *a* becomes *a*(*t*).

$\dot{D} = \dot{a} \cdot \Delta x = \dot{a} \cdot \frac{a}{a} \cdot \Delta x = H \cdot D = V$, where H is the Hubble constant. The Hubble constant does not depend on position, but it does depend on time. The further apart are the galaxies, the faster they are separating. There is no center because every galaxy is moving away from every other galaxy.

Hubble used the size and relative brightness of starts to estimate distance. He made a scatter plot of V versus D and found that there was a roughly linear relationship between the two. Initially the estimate of V was ten times too big. In Hubble's day the galaxies were not known. Today we use the Doppler shift of the light emitted by the galaxy to determine its distance. Consider now a plane with equally spaced galaxies.



$$D = \sqrt{a^2 \cdot \Delta x^2 + a^2 \cdot \Delta y^2} = a^2 \cdot \sqrt{\Delta x^2 + \Delta y^2}$$

$$V = \dot{a} \cdot \frac{a}{a} \cdot \sqrt{\Delta x^2 + \Delta y^2} = D \cdot \frac{\dot{a}}{a}$$ Hubble's Law is $V = D \cdot H$.

The geometry is determined by the metric.

$$ds^2 = a^2 \cdot dx^2 \qquad d\tau^2 = dt^2 - \frac{a(t)^2}{c^2} \cdot dx^2 \qquad g = \begin{pmatrix} 1 & 0 \\ 0 & -a^2 \end{pmatrix}$$

In three dimensions.

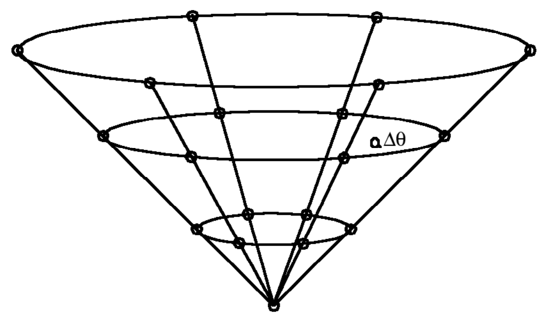
$$d\tau^2 = dt^2 - \frac{a^2}{c^2} \cdot dx^2 \ldots \qquad g = \frac{1}{c^2} \cdot \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -a^2 & 0 & 0 \\ 0 & 0 & -a^2 & 0 \\ 0 & 0 & 0 & -a^2 \end{pmatrix}$$

Space is homogeneous $a \neq a(x, y, z)$ and isotropic $a_x = a_y = a_z$.

As an example of closed space, consider life on a circle of circumference $2\cdot\pi\cdot a$, with galaxies $\Delta\theta$ equally spaced by an arc length of a that may change with time. The metric is

$$ds^2 = a(t)^2 \cdot d\theta^2 \qquad d\tau^2 = dt^2 - \frac{a(t)^2}{c^2}\cdot d\theta^2 \ .$$

$$D = a\cdot\Delta\theta \qquad V = \frac{\dot{a}}{a}\cdot D$$



The figure above shows *a* increasing linearly with time (upward) to give a cone. If *a* were not a function of time, then the figure would be a cylinder. The parameter *a* could have any time dependence, which would make the shape more complicated. So the Big Bang is not an explosion but rather Hubble's law with a time-dependent Hubble constant.

End lecture #1